

Gaussian half-duplex cooperative compression for relay channels

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Abstract

We propose compressive transmission, which uses CS as the channel code and directly transmits multi-level CS random projections through amplitude modulation, inspired by the CS theory and its strong association with low-density parity-check code. Compressive collaboration mechanisms inside a relay channel are the topic of this essay. In this study, we examine and quantify the possible rates of four decode-and-forward (DF) techniques in a three-terminal half-duplex Gaussian relay channel: receiver diversity, code diversity, consecutive decoding, and concatenated decoding. Numerical computation and simulated experiments are used to evaluate the four different plans. We also analyse a different source channel coding strategy for transmitting sparse sources and compare it to compressive collaboration. Compressive collaboration has significant promise in terms of transmission efficiency and channel adaptability.

Introduction

An up-and-coming field of study, "compressive sensing" (CS) [1,2] deals with the capture and recovery of sparse signals using a modest number of randomly chosen linear projections. Recently, it has been noticed that LDPC codes, a well-known kind of channel coding, are strongly connected to CS [3,4]. More specifically, the CS reconstruction technique presented by Baron et al. [5] is almost similar to Lucy's LDPC decoding algorithm [6] when the measurement matrix in CS is used as the parity-check matrix of an LDPC code. Since CS codes are comparable to LDPC codes, we propose and investigate compressive transmission, which employs CS as the channel code and directly transmits multi-level CS random projections through amplitude modulation. CS may be thought of as a combined source-channel code and channel-protection code due to its source-compression and channel-protection features. Conventional schemes employ source coding to compress data before adopting channel coding to secure the compressed data across the lossy channel while transmitting sparse or compressible data. Compressive transmission has several distinct benefits over such a standard approach. At the transmitter end, CS simplifies things since it employs random projections to create measurements independent of the compressible patterns. This is useful for sensor nodes and other thin signal gathering devices like single-pixel cameras [7]. Additionally, it strengthens durability. Compressed data are notoriously vulnerable to the smallest of bit

mistakes. When the channel code is inadequate to safeguard data in an unexpectedly degraded channel, the whole coding block or perhaps the entire data sequence may become undecodable under the traditional method. CS random projections, on the other hand, directly act on

source bits, therefore random bit mistakes have no effect on the quality of the data as a whole.

Model of a Channel

Here, we focus on a three-endpoint relay channel with half-duplex operation [9]. S, R, and D stand for "source," "relay," and "destination," respectively. Let's call c_{sd} , c_{sr} , and c_{rd} the channel gains of three direct connections (S, D), (S, R), and (R, D). In this study, we assume that the relay is situated along the SD line, with equidistant travel times between the two ends. Attenuating by a factor of 2, the channel gains are $c_{sd} = 1$, $c_{sr} = c_{rd} = 4$, and $c_{rd} = c_{sd} = 4$. In a half-duplex setup, the relay R is only allowed to receive, not broadcast. So, as shown in Figure 1, the channel is time-shared between broadcast (BC) mode and multiple access (MAC) mode. If we designate BC mode's time percentage as t ($0 < t < 1$), then MAC mode's time proportion is $1-t$. The source sends out symbol x_1 while operating in BC mode. The relay and the receiver may both pick up the sound. Both the relay and the final destination have picked up signals of y_r and y_{d1} , respectively.

$$y_r = \sqrt{c_{sr}}x_1 + z_r$$
$$y_{d1} = \sqrt{c_{sd}}x_1 + z_{d1}$$

where z_r and z_{d1} are Gaussian noises perceived at R and D

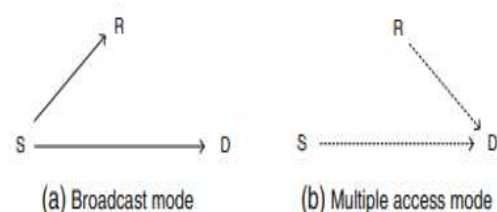


Fig 1 A Three – terminal relay network with R operating in half – duplex mode

At the end of BC mode, the relay generates message w based on its received signals. Then in MAC mode, the source transmits x_2 while the relay transmits w simultaneously. The destination receives the superposition of the two signals which can be represented by:

$$y_{d2} = \sqrt{c_{sd}}x_2 + \sqrt{c_{rd}}w + z_{d2}$$

where z_{d2} is the perceived Gaussian noise at D. Finally, the destination D decodes original message from received signals during BC and MAC modes. Assume that random variables Z_r , Z_{d1} and Z_{d2} , corresponding to the noises z_r , z_{d1} and z_{d2} , have the same unity energy. Thus, the system resource can be easily characterized by the transmission energy budget E . Denote E_{s1} , E_{s2} and E_r as the average symbol energy for random variables X_1 , X_2 and W , which correspond to x_1 , x_2 and w , respectively. Then the system constraint can be described by the following inequality:

$$tE_{s1} + (1 - t)(E_{s2} + E_r) \leq E$$

For clarity of presentation, the following notations are defined as the received signal strength at different links:

$$\begin{aligned} P_{sd1} &= c_{sd}E_{s1}, & P_{sr} &= c_{sr}E_{s1} \\ P_{sd2} &= c_{sd}E_{s2}, & P_{rd} &= c_{rd}E_r \end{aligned}$$

Compressive transmission overview 3.1
Compressive transmission in a relay channel in this research, we model the source data as i.e., bits with probability p to be 1 and probability $1 - p$ to be 0. When $p = 0.5$, the source is considered sparse or compressible. During transmission, source bits are segmented into length- n blocks. Let $\mathbf{u}=[u_1, u_2, \dots, u_n]$ be one source block. In order to transmit \mathbf{u} over the relay channel, the source first generates CS measurements using a sparse Rademacher matrix with elements drawn from $\{0, 1, -1\}$, and transmits them in the BC mode. The transmitted symbols, which consist of m_1 measurements, can be represented by:

$$\mathbf{x}_1 = \sqrt{\alpha_{s1}}A_1\mathbf{u}$$

where α_{s1} is a power scaling parameter to match with sender's power constraint

In MAC mode, the source generates and transmits another m_2 measurements using identical/different Rademacher matrix, which can be represented by:

$$\mathbf{x}_2 = \sqrt{\alpha_{s2}}A_2\mathbf{u}$$

This article studies DF strategies and leaves compressand-forward (CF) strategies to future research. A prerequisite of DF relaying is that the relay can fully decode the messages transmitted by the source in BC mode. With this assumption, the relay can generate new measurements of \mathbf{u} and transmits them in MAC mode:

$$\mathbf{w} = \sqrt{\alpha_r}B\mathbf{u}$$

where B is also a Rademacher matrix, and \mathbf{w} contains m_2 measurements. The power scaling parameters in above equations ensure that:

$$E[X_1^2] \leq E_{s1}; \quad E[X_2^2] \leq E_{s2}; \quad E[W^2] \leq E_r$$

Under these power constraints, the corresponding scaling parameters α_{s1} , α_{s2} and α_r can be derived, where the average power of symbol $A_1\mathbf{u}$, $A_2\mathbf{u}$ and $B\mathbf{u}$ are determined by the row weight of corresponding sampling matrix and sparsity probability of \mathbf{u} . Since m_1 measurements are transmitted in BC mode and m_2 measurements are transmitted in MAC mode, the time proportion of BC mode can be calculated as:

$$t = \frac{m_1}{m_1 + m_2}$$

The destination will perform CS decoding from all the measurements received in both modes. The belief propagation algorithm (CS-BP) proposed by Baron et al. [5] is adopted in our system. If the decoding is successful, the transmission rate can be computed by:

$$R = \frac{H(\mathbf{u})}{m_1 + m_2}$$

where $H(\mathbf{u})$ is the entropy of \mathbf{u} and m_1 and m_2 determines the cost, time slots for the BC mode and MAC mode, respectively. If the base of the logarithm in entropy computation is 2, the rate R is expressed in bits per channel use. It should be noted that the rate R in Equation (11) is related with the symbol energy E_{s1} , E_{s2} and E_r . For the compressive transmission along a link channel, when the corresponding transmission power is larger, higher quality of measurements could be derived and the number of needed measurements for source recovery could be smaller. Therefore, higher rate could be achieved from large allocated transmission energy. In such compressive transmission system, the encoding complexity is rather low because the calculation of measurements at the source node only involves the sums and differences of a small subset of the source vector.

The complexity of the belief-propagation based decoding algorithm is $O(TMLQ \log(Q))$ [5], where L is the average row weight, Q is the dimension of transmitted message in belief propagation process, T is the iteration number and M is the number of received measurements.

Numerical study and simulations

In the previous section, we have proposed four DF schemes and formulated their achievable rates. In this section we will first evaluate the four compressive cooperation strategies through both numerical studies and MATLAB simulations, and then comparison between compressive transmission and a conventional scheme based on source compression and binary channel coding is made. In both evaluations, the binary source message with $p = 0.1$ is considered. As the source is binary, we can evaluate the channel rate with bit rate and characterize the unperfect transmissions with bit error rate (BER). For convenience, instead of information rate we present the results using bit rate:

$$R_b(P) = n/(m_1 + m_2)$$

where n is the block length of u . We set $n = 6000$ if not otherwise stated. All the results shown in this section are about $R_b(P)$. However, we continue to use notation $R(P)$ when the statement is valid for both rates. Actually, for 0.1-sparse data, the bit rate $R_b(P)$ differs from the information rate $R(P)$ (11) only by a constant coefficient:

$$R(P) = H(p = 0.1) \times R_b(P) \approx 0.469 \times R_b(P)$$

At the end of Section 3, we introduce the notion $R((\gamma_1, P_1), \dots, (\gamma_k, P_k))$ to denote the achievable rate when CS measurements are received from multiple channels. This creates an additional dimension in characterizing channel rates. Without reasonable simplification, we will be unable to compute the optimal rates of different DF schemes even through numerical integration. Therefore, we approximate the achievable rate of combined channels with:

$$R((\gamma_1, P_1), \dots, (\gamma_k, P_k)) \approx \sum_i \gamma_i R(P_i)$$

This approximation is reasonable because otherwise a source needs to do per measurement energy allocation to achieve the optimal performance.

Evaluating compressive cooperation strategies

In the formulation of the proposed four DF schemes, the supremum is taken over all possible time proportion and transmission powers that satisfy (4). The analytical solution to the optimization problem is hard to find since $R(P)$ is unknown. Therefore, we first obtain $R(P)$ for compressive transmission through simulations, and then compute the achievable rates of the four DF strategies through numerical integration. Baron et al. [5] have reported that there is an optimal row weight $L_{opt} \approx 2/p$ beyond which any performance gain is marginal. We slightly adjust L to 15 and use eight -1 's and seven 1 's. For simplicity, we use the amplitude modulation of only one carrier wave. The performance for quadrature amplitude modulation (QAM) can be easily deduced from our reported results. Figure 3 shows the achievable rates of the four DF schemes as well as direct transmission. The four schemes are denoted by codd (code diversity), recd (receiver diversity), succ (successive decoding), and conc (concatenated decoding). It is observed that transmitting through a relay greatly increases channel throughput when channel SNR is low and the benefit is not significant when SNR is higher than 15 dB

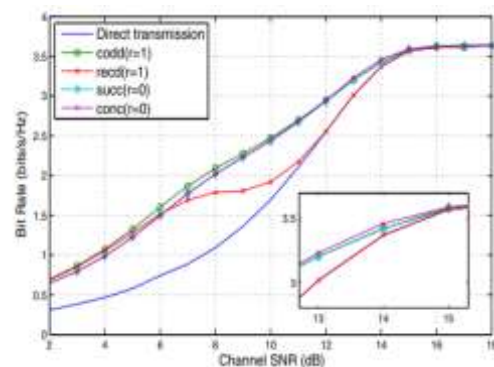


Figure 3 Comparing the bit rates of different DF schemes.

shape when the x-axis is plotted in dB, it is a concave function with respect to P . Considering that $R(0) \geq 0$, $R(\cdot)$ is subadditive, i.e.

$$R(P_1) + R(P_2) \geq R(P_1 + P_2)$$

Using this property, it can be derived that the rate of receiver diversity is no greater than that of code diversity. The comparison between the code diversity scheme for $r = 1$ and the two $r = 0$ schemes draws a consistent conclusion as in conventional relay channels. First of all, the performance difference between $r = 0$ schemes and $r = 1$ schemes is not significant. Second, $r = 0$ schemes show advantage when channel SNR is high, but $r = 1$ schemes perform better when SNR is low. Our numerical results show that the achievable rate of $r = 0$ schemes is higher than $r = 1$

schemes when SNR is higher than 13 dB. Although the two $r = 0$ schemes exhibit similar performance, concatenate decoding appears to be better than successive decoding when channel SNR is higher than 13 dB. We next carry out simulations to evaluate the gap between real implementations and numerical computations. The simulations are performed through the following process. First, the optimized parameters, including time proportion and energy allocation, are retrieved from the numerical study for all three schemes. Then, average BER is measured through a set of test runs. If the BER is larger than 10^{-5} , which is considered as the threshold of reliable transmission, we increase channel SNR until the BER goes below 10^{-5} . This SNR-rate pair is plotted on Figure 4. In Figure 4, simulation results of three DF schemes are compared with the highest numerical rate computed when r is either 0 or 1. It can be seen that the implementation gap is within 1.4 dB for all three schemes. During simulation, we observe that code diversity has very stable performance at both high and low SNRs. The performance of the two $r = 0$ schemes has a slightly larger variation. In addition, when channel SNR is lower than 12 dB, both $r = 0$ schemes degrade to two-hop transmission, i.e. $E_s = 0$. Considering the fact that $r = 0$ schemes do not significantly improve channel rate at high SNR, and code diversity is easier to implement, it is a wise choice to stick to code diversity scheme in practical systems.

We also evaluate the BER performance of compressive cooperation. Because the three DF schemes have very similar BER performance, we only present the results of code diversity scheme in Figure 5. The target rates of the five curves are computed at 6, 8, 10, 12, and 14 dB, respectively. For each target rate and its computed optimal parameters, we slightly vary the channel SNR and evaluate the average BER. An interesting finding from the figure is that the BER of compressive cooperation does not steeply increase when the channel condition decreases from the channel SNR that ensures reliable transmission. It is in sharp contrast to conventional coding and modulation schemes whose typical BER curves can be seen in Figure 6. This special BER property suggests that compressive transmission is more robust for highly dynamic channels where precise channel SNR is hard to obtain. Actually, when wireless channel state information is unknown for the source node, the channel code based on CS measurements can be generated limitlessly and

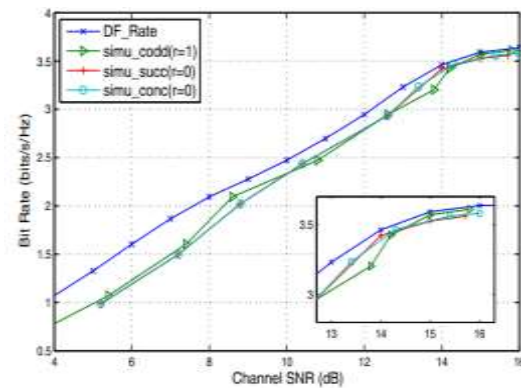


Figure 4 Simulation results of three DF schemes.

sent until the recipient reaches a certain degree of recovery. As illustrated in [5], increasing the number of CS measurements will provide additional redundancy, which will assist overcome the channel noise. Compressive cooperative communication systems benefit greatly from this rateless quality in comparison to conventional LDPC codes in their ability to adapt to channel fluctuation. In this section's last section, we examine and contrast the computational complexity of the four DF approaches.

Conclusion

Using CS random projections as the combined source-channel code, this article suggests a method of compressive transmission. In this paper, we introduce a three-terminal half-duplex Gaussian relay network and define and assess four DF cooperative techniques for compressive transmission. The possible rates of these techniques are evaluated using both numerical research and simulated exercises. We've compared the compression ratio of compressive collaboration to that of a more traditional coding strategy for distinct source channels. High transmission efficiency and good channel adaptation are only two reasons why the suggested compressive collaboration shows promise in the wireless relay channel.

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